（平成30年度一般後期）　　数　学

1

(1) 210 組.

(2) 40 個.

(3) 8 個.

(4) 50 通り.

(5) 42 通り.

2

(1) $\vec{a}\cdot\vec{b} = 7$

(2) $\overrightarrow{\mathrm{OH}} = \frac{7}{16}\left(\vec{b}+\vec{c}\right)$　　$\overrightarrow{\mathrm{OP}} = \frac{7}{23}\left(\vec{a}+\vec{b}+\vec{c}\right)$

$\overrightarrow{\mathrm{OQ}} = \frac{7}{16}\left(\vec{a}+\vec{b}\right)$

(3) 四面体 OBCQ の体積は $\frac{7}{48}\sqrt{23}$

3 （最後の答だけでなく，答の導き方も書くこと。）

(1) $y' = 3x^2 - 6x$ より，$x = a_{n+1}$ での接線は，

$\ell_n : y = (3a_{n+1}^2 - 6a_{n+1})(x - a_{n+1}) + a_{n+1}^3 - 3a_{n+1}^2$

$= (3a_{n+1}^2 - 6a_{n+1})x - 2a_{n+1}^3 + 3a_{n+1}^2$

$x^3 - 3x^2 = (3a_{n+1}^2 - 6a_{n+1})x - 2a_{n+1}^3 + 3a_{n+1}^2$

$(x - a_{n+1})^2(x + 2a_{n+1} - 3) = 0$

$a_n \neq a_{n+1}$ より，$a_n = -2a_{n+1} + 3$

$a_{n+1} = -\frac{1}{2}a_n + \frac{3}{2}$ より，$a_{n+1} - 1 = -\frac{1}{2}(a_n - 1)$ から，

$a_1 = 3$ であるので，$\underline{a_n = 2\cdot\left(-\frac{1}{2}\right)^{n-1} + 1}$

(2) $\alpha_n = 3a_{n+1}^2 - 6a_{n+1} = 3a_{n+1}(a_{n+1} - 2)$

(1) より，$a_{n+1} = 2\cdot\left(-\frac{1}{2}\right)^n + 1$ であるので，

$\alpha_n = 3\cdot\left\{2\cdot\left(-\frac{1}{2}\right)^n + 1\right\}\left\{2\cdot\left(-\frac{1}{2}\right)^n - 1\right\} = \underline{3\cdot\left(\frac{1}{4}\right)^{n-1} - 3}$

(3) $a_n < a_{n+1}$ のとき

$S_n = \int_{a_n}^{a_{n+1}} \left\{x^3 - 3x^2 - (3a_{n+1}^2 - 6a_{n+1})x + 2a_{n+1}^3 - 3a_{n+1}^2\right\} dx$

$= \int_{a_n}^{a_{n+1}} (x - a_n)(x - a_{n+1})^2 dx = \frac{1}{12}(a_{n+1} - a_n)^4$

$a_n > a_{n+1}$ のとき

$S_n = \int_{a_{n+1}}^{a_n} \left\{(3a_{n+1}^2 - 6a_{n+1})x - 2a_{n+1}^3 + 3a_{n+1}^2 - (x^3 - 3x^2)\right\} dx$

$= \int_{a_{n+1}}^{a_n} -(x - a_n)(x - a_{n+1})^2 dx = \frac{1}{12}(a_{n+1} - a_n)^4$

$a_{n+1} - a_n = 2\cdot\left(-\frac{1}{2}\right)^n + 1 - \left\{2\cdot\left(-\frac{1}{2}\right)^{n-1} + 1\right\} = -3\cdot\left(-\frac{1}{2}\right)^{n-1}$

ゆえに，$S_n = \frac{1}{12}\cdot(-3)^4\cdot\left\{\left(-\frac{1}{2}\right)^{n-1}\right\}^4 = \underline{27\cdot\left(\frac{1}{4}\right)^{2n-1}}$